New Approaches to Teaching Linear Algebra
Abstract

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0.1 Problem Statement

Linear Algebra (Math 310) is a course which is required of all Mathematics and Engineering majors at VCU. The course is partly computational and partly an abstract investigation of properties of vector spaces. For the engineering students in particular this part of the course is a great change from the largely algorithmic mathematics that they met in their earlier courses. Many of the objects that the engineers encounter in their mathematical practice, polynomials, matrices, functions, n-tuples, etc., fall under the unifying framework of a vector space—and anything that is true about a vector space in general is true about these objects.

0.2 Aims

The instructor has taught in the traditional lecture format throughout his career. One of the aims of this proposal was to learn alternative classroom approaches. The instructor attended two regional meetings and one national meeting as a member of Project NeXT (New Experiences in Teaching) and met, engaged and networked with a large number of successful teachers and learned many ideas for improving student learning and success.

The primary aim of the proposal was to develop new course materials for teaching Linear Algebra. A series of worksheets was developed for daily classroom assignments. In the traditional lecture students passively listen and take notes. They often leave believing that they understand the material but, when they subsequently attempt the homework problems, they discover that their "understanding" has vanished. In mathematics, the test of understanding is the ability to work problems. In the traditional lecture classroom style students are not asked to test their understanding of the material that is being presented.

0.3 New Course Materials

A more recent trend is for active learning classroom styles. On this model, students in some way engage with the material that is being presented. In the presented implementation, students are engaged with worksheets. A prototype example of a central concept or problem type is presented, and then the students are asked to work a similar problem on their worksheet.

The worksheets are not quizzes. There is no pressure. Students are allowed and encouraged to talk to each other. The instructor walks around the classroom both to gauge how successfully the students are doing and how well they understand the presented concepts, but also to help. This is one more teaching opportunity for the instructor.

The worksheets are collected and recorded as part of an in-class assignment component of their semester grade. Everyone who was in class and worked on the worksheet receives full credit. The worksheet problems are chosen both to reinforce the lecture but also to preview the homework. Furthermore test questions are chosen that reflect both of these. Students have ample opportunity to master the new concepts and they know exactly what is expected of them.

0.4 Results

The Fall 2009 semester is the first semester that this approach is being tested. In the first place, the students seem to enjoy taking a break form the lecture to work on the worksheets. In the past, students could tune out when they encountered vector spaces or any other seemingly difficult abstractions. In the described approach, they can no longer do this. By filling out worksheets they are required to engage. Vector spaces have already been introduced—there were five worksheets on this topic alone. The students' understanding seems much deeper than in past semesters. There is a trade-off: some teaching time is lost. In this case the pros easily outweigh the cons. The first semester grades are forthcoming as is grade comparison data with past semesters. End-of-semester exit surveys of student opinion of the worksheets is also forthcoming.